PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

[Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Missouri.]

PROBLEMS FOR SOLUTION.

2806. Proposed by WARREN WEAVER, Throop College of Technology.

The Maxwell law for the distribution of velocities among the molecules of a gas shows that, considering a velocity of given magnitude, it is equally probable that it have any direction. Find an expression for the probability that the velocity of a molecule m_2 (whose velocity relative to the mass motion of the gas is k') when measured relative to the velocity k of a molecule m_1 (k less than k') shall make with k an angle falling within the limits θ and $\theta + d\theta$.

2807. Proposed by S. A. COREY, Des Moines, Iowa.

Establish the identity

$$\begin{vmatrix} -x & bcy & -acu & -abv \\ y & x & -av & au \\ u & bv & x & by \\ -v & cu & cy & -x \end{vmatrix}^2 + ac\begin{vmatrix} x & bcy & -x & -abv \\ -y & x & y & au \\ u & bv & u & by \\ -v & cu & -v & -x \end{vmatrix}^2$$

$$+ ab\begin{vmatrix} x & bcy & -acu & -x \\ -y & x & -av & y \\ u & bv & x & u \\ -v & cu & cy & -v \end{vmatrix}^2 = \begin{vmatrix} x & bcy & -acu & -abv \\ -y & x & -av & au \\ u & bv & x & by \\ -v & cu & cy & -x \end{vmatrix}^2$$

2808. Proposed by A. A. BENNETT, University of Texas.

Let $f(x)$ denote an arbitrary frequency distribution function.

(1) Show that the condition that frequencies when compounded follow the law for simple frequencies is that the iteration of $f(r - t)$ in the sense of integral equations leaves this kernel unaltered, *i.e.*,

(i) $f(r - t) = \int_{-\infty}^{+\infty} f(r - s)f(s - t)ds$ and, hence, this kernel is singular.

Cf. David F. Barrow, *Annals of Mathematics*, second series, vol. 19 (1917), p. 97.

(2) Show that $f(a, x)$ satisfies (i) for every real value of a, when

$$f(a, x) = \begin{cases} \dfrac{1}{\pi x}\sin ax, \text{ for } -c < x < c, c \text{ being arbitrary, real and positive but fixed. In particular } c \text{ may be infinite.} \\ 0, \begin{cases} \text{for } x < -c. \\ \text{for } x > c. \end{cases} \end{cases}$$

(3) Show that for a given c, and $0 < a < b$, $f(b, x) - f(a, x)$ and $f(a, x)$ are orthogonal.

(4) Show that the Gaussian normal probability function satisfies (i). Consider its expansion in terms of the orthogonal functions of (3).

(5) Show that for c finite, $f(a, x)$ cannot be defined by means of a Fourier series, save for discrete values of a, since for a varying within certain restricted segments, the Fourier coefficients of $f(a, x)$ remain unaltered. Show that when a Fourier representation is possible, the series reduces to a trigonometric polynomial.

2809. Proposed by the late L. G. WELD.

Find the nth term of the series defined by the relation, $u_{i+2} = u_i + u_{i+1}$, in which $u_1 = u_2 = 1$.

2810. Proposed by H. S. UHLER, Yale University.

In the expansion of the following determinant or eliminant, find the total number of terms, and the number of terms having the coefficients + 1, − 1, + 2, − 2, + 3, − 3, + 4, − 4, + 5, − 5, + 6, − 8, + 10, respectively.

$$\begin{vmatrix} a & b & c & d & e & 0 & 0 & 0 \\ 0 & a & b & c & d & e & 0 & 0 \\ 0 & 0 & a & b & c & d & e & 0 \\ 0 & 0 & 0 & a & b & c & d & e \\ A & B & C & D & E & 0 & 0 & 0 \\ 0 & A & B & C & D & E & 0 & 0 \\ 0 & 0 & A & B & C & D & E & 0 \\ 0 & 0 & 0 & A & B & C & D & E \end{vmatrix}$$

2811. Proposed by J. L. RILEY, Stephenville, Texas.

Given the cube roots of 60, 61, 63, and 64, to find the cube root of 62 by the method of differences.

2812. Proposed by C. N. SCHMALL, New York City.

If $F(x, y, z)$ be a homogeneous function of x, y, z, which becomes $\phi(u, v, w)$ by the elimination of x, y, z, by means of the equations $\partial F/\partial x = u$, $\partial F/\partial y = v$, $\partial F/\partial z = w$; show that

$$\frac{\partial F}{\partial u}\Big/x = \frac{\partial F}{\partial v}\Big/y = \frac{\partial F}{\partial w}\Big/z.$$

2813. Proposed by PAUL CAPRON, U. S. Naval Academy.

An ellipse having the major-axis $2a$ and the eccentricity ϵ, is revolved first about its major axis, forming a prolate spheroid, then about its minor axis forming an oblate spheroid. Show that the surfaces of these spheroids are, respectively,

$$2\pi a^2(1/\epsilon\sqrt{1-\epsilon^2}\sin^{-1}\epsilon + 1)$$

and

$$2\pi a^2\left[2 + 1/\epsilon(1-\epsilon^2)\log\left(\frac{1+\epsilon}{1-\epsilon}\right)\right].$$

SOLUTIONS OF PROBLEMS.

339 (Calculus) [June, 1913; May, 1919]. Proposed by T. H. GRONWALL, Washington, D. C.

To show that for any real value of x

$$\left|\frac{d^n}{dx^n}\left(\frac{\sin x}{x}\right)\right| \leqq \frac{1}{n+1}, \quad \text{and} \quad \left|\frac{d^n}{dx^n}\left(\frac{1-\cos x}{x}\right)\right| \leqq \frac{1}{n+1}.$$

I. Solution by Otto Dunkel, Washington University.

The function $y = \sin x/x$, $x \neq 0$; $y = 1$, $x = 0$, is single-valued and continuous for all values of x and it can be expressed as a power series

$$y = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots.$$

The power series shows that the function possesses all of its derivatives at $x = 0$, and it is easily seen from the power series or from the original form of the function that the derivatives exist for all other values of x. Hence, we may write the sequence of equations for finding the derivatives

$$xy = \sin x, \qquad y + xy' = \sin\left(\frac{\pi}{2} + x\right),$$

$$2y' + xy'' = \sin(\pi + x), \qquad \cdots, \qquad (n+1)y^{(n)} + xy^{(n+1)} = \sin\left(\frac{n+1}{2}\pi + x\right), \qquad \cdots$$